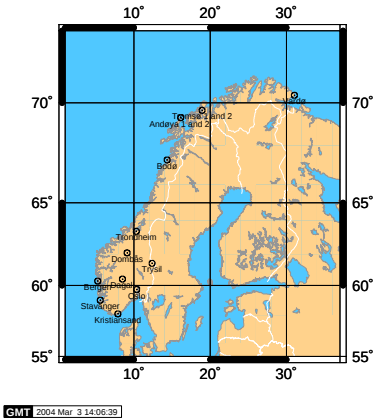


Principal Component Analysis of the Norwegian permanent GPS stations

Trond Arve Haakonsen ⁽¹⁾, Hossein Nahavandchi ⁽¹⁾, Hans-Peter Plag ⁽²⁾
(1) Department of Geomatics, Norwegian University of Science and Technology, 7491,Trondheim
(2) Geodetic Institute, Norwegian Mapping Authority.



INTRODUCTION:

Time series of permanent GPS-stations are increasingly used to determine displacements of points at Earth's surface for practical and scientific purposes. During the last years, linear models used for univariate time series have essentially been improved. A natural continuation is to analyse several series together, as the multivariate time series analysis introduced in this poster. Very small correlations between time series of residuals for the 3D-coordinates of each site are found, but some large correlations between respective directions for different sites have been revealed. An attempt to demonstrate the significant covariation or common fluctuation for all sites using the theory of Principal Component Analysis (PCA), also known as Empirical Orthogonal Functions (EOF) or Common Modes, will be done. Time series from 14 permanent Norwegian GPS stations (SATREF) are used in this investigation. From epochs of 30 seconds, 3D-solutions representing daily means are processed using the GIPSY-OASIS-II-software ⁽¹⁾ according to the precise point positioning method of [Zumberge et. al., 1997]. Standard deviations of daily means are computed to indicate accuracies making a basis for weighting in the further analysis. The time series are of different lengths and not either stationary or complete (a lot of gaps occur). As a result of the Least-squares spectral analysis, see [Haakonsen et. al., 2003], significant frequency components of one, two and four cycles a year are revealed. To make the series from several sites comparable, multiple weighted linear regressions have been performed for each time series, and parameters for all known constituents as shifts, linear and periodic terms estimated. The choice of deterministic models has shown to be of vital importance. The adequacy of the models is tested by normality, blunders and outliers tests for the time series of normalised residuals. As an input to the PCA, correlation matrices for each direction, estimated from the series of normalised residuals, are preferred to ensure equal weighting of all stations and to compensate for series of different lengths. To overcome the problem of gaps in data, an alternative correlation technique, using valid combinations, representing only common epochs from two data series are used. The method also ensures the correlation matrix to be positive semi definite.

Large correlations indicate that the data, without much loss of information, possibly may be replaced by a set of fewer variables. A new set of variables is searched (under some constraints) as uncorrelated linear combinations of the original variables. The new variables are sorted in descending order after the magnitude of their variances and denoted as Principle Components (PC). The PC's may be found from a singular value or a spectral decomposition of the correlation matrix. The linear relations between original and new variables are found as coefficients in a square matrix. Under the constraints that the column vectors should be of unit length, the square matrix will consist of the normalised eigenvectors of the correlation matrix. The variances of the PC's are identical to the eigenvalues of the correlation matrix: λ_j ($j=1,2,...,p$), which are all positive, provided that the square symmetric correlation matrix is positive definite. The proportion of total population variance explained by the j th PC is then λ_j/p . Each component e_{jk} of the normalised eigenvectors e_j is a measure of the importance of the k th variable to $PC(j)$, irrespective of the other variables. Alternatively, scaled versions of the normalised eigenvectors, denoted *factor loadings* defined as: $l_j = \sqrt{\lambda_j} e_j$ are used to visualise in which way a few possible *common factors* influences the data.

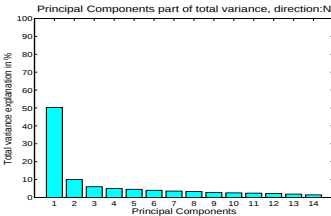
NUMERICAL INVESTIGATIONS:

Table 1: Correlation matrices

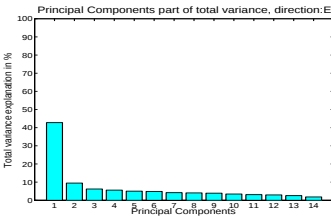
Correlation matrices computed from time series of normalised residuals for North, East and Height directions																																											
No	Station	series	North												East												Height																
1	Andoya 1	ans1	1.00	0.33	0.34	0.42	0.38	0.17	0.31	0.39	0.38	0.37	0.36	0.22	0.41	0.30	0.28	0.36	0.31	0.37	0.36	0.33	0.29	0.34	0.31	1.00	0.41	0.38	0.45	0.29	0.26	0.25	0.23	0.28	0.34	0.36	0.38	0.39	0.29				
2	Andoya 2	ans2	0.33	1.00	0.42	0.68	0.39	0.33	0.43	0.43	0.46	0.52	0.53	0.42	0.49	0.52	0.39	0.68	0.39	0.44	0.54	0.42	0.38	0.31	0.54	0.20	1.00	0.49	0.48	0.50	0.47	0.42	0.40	0.45	0.47	0.46	0.44	0.52					
3	Bergen	ber3	0.34	0.42	1.00	0.53	0.53	0.51	0.66	0.60	0.73	0.55	0.37	0.31	0.69	0.33	0.32	0.35	1.00	0.45	0.52	0.43	0.45	0.44	0.54	0.42	0.38	0.31	0.54	0.20	0.45	0.47	0.42	0.40	0.45	0.48	0.52	0.26					
4	Bodø	bod5	0.42	0.68	0.53	1.00	0.38	0.46	0.50	0.53	0.50	0.52	0.50	0.48	0.62	0.53	0.42	0.43	0.45	1.00	0.45	0.52	0.43	0.45	0.44	0.54	0.42	0.38	0.31	0.54	0.20	0.45	0.47	0.42	0.40	0.45	0.48	0.52	0.26				
5	Dagali	dag3	0.38	0.39	0.53	0.38	1.00	0.38	0.52	0.39	0.58	0.44	0.36	0.21	0.59	0.38	0.24	0.38	0.52	0.42	1.00	0.48	0.49	0.29	0.54	0.49	0.31	0.28	0.44	0.24	0.27	0.40	0.39	0.38	0.36	0.38	0.24	0.25	0.51				
6	Dønnes	dnn3	0.17	0.33	0.51	0.46	0.38	1.00	0.47	0.42	0.56	0.51	0.33	0.25	0.49	0.30	0.24	0.38	0.52	0.42	0.48	1.00	0.48	0.49	0.29	0.54	0.49	0.31	0.28	0.44	0.24	0.27	0.40	0.39	0.38	0.36	0.38	0.24	0.25				
7	Kristiansund	kri3	0.31	0.43	0.66	0.55	0.52	0.47	1.00	0.66	0.75	0.56	0.40	0.33	0.66	0.36	0.36	0.36	0.45	0.44	0.38	0.68	1.00	0.43	0.55	0.42	0.38	0.34	0.52	0.20	0.25	0.25	0.27	0.47	0.41	0.24	0.44	0.34	0.59	0.29			
8	Mosjøen	mos3	0.38	0.43	0.69	0.53	0.39	0.42	0.66	0.55	0.55	0.33	0.31	0.57	0.36	0.53	0.33	0.33	0.33	0.44	0.44	0.38	0.68	0.43	0.55	0.42	0.38	0.34	0.52	0.20	0.25	0.25	0.27	0.47	0.41	0.24	0.44	0.34	0.59	0.29			
9	Steinkjer	ste3	0.39	0.46	0.73	0.59	0.58	0.56	0.75	0.65	1.00	0.81	0.43	0.33	0.73	0.38	0.54	0.47	0.54	0.43	0.55	0.45	0.69	0.45	0.59	0.36	0.29	0.36	0.52	0.21	0.21	0.31	0.28	0.38	0.43	0.34	0.48	0.46	0.42				
10	Trondheim	trn3	0.37	0.52	0.55	0.62	0.44	0.51	0.56	0.55	0.61	1.00	0.41	0.35	0.66	0.45	0.51	0.33	0.45	0.38	0.46	0.31	0.22	0.38	0.28	0.29	0.31	0.18	0.66	0.66	0.34	0.27	0.36	0.38	0.41	0.41	0.40	0.42					
11	Trondheim 2	trn2	0.38	0.53	0.57	0.58	0.36	0.33	0.44	0.33	0.43	0.41	1.00	0.59	0.45	0.51	0.33	0.45	0.38	0.46	0.31	0.22	0.38	0.28	0.29	0.31	0.18	0.66	0.66	0.34	0.27	0.36	0.38	0.41	0.41	0.40	0.42						
12	Tromsø 1	tro3	0.22	0.42	0.31	0.46	0.21	0.25	0.33	0.31	0.33	0.35	0.50	1.00	0.30	0.45	0.33	0.46	0.34	0.49	0.34	0.27	0.53	0.47	0.53	0.52	0.34	0.31	1.00	0.32	0.39	0.41	0.41	0.41	0.41	0.41	0.41						
13	Trøyli	trt3	0.41	0.49	0.69	0.62	0.59	0.49	0.66	0.57	0.75	0.66	0.45	0.39	1.00	0.42	0.45	0.35	0.46	0.49	0.44	0.27	0.53	0.47	0.53	0.52	0.34	0.31	1.00	0.32	0.39	0.41	0.41	0.41	0.41	0.41	0.41						
14	Varde	var3	0.35	0.52	0.33	0.53	0.38	0.30	0.36	0.36	0.37	0.42	0.51	0.45	0.42	1.00	0.35	0.38	0.48	0.48	0.34	0.27	0.53	0.47	0.53	0.52	0.34	0.31	1.00	0.32	0.39	0.41	0.41	0.41	0.41	0.41	0.41						
Average correlation:			0.33	0.45	0.51	0.52	0.41	0.40	0.52	0.47	0.55	0.51	0.42	0.34	0.55	0.48	0.32	0.39	0.49	0.44	0.39	0.34	0.40	0.37	0.43	0.41	0.35	0.35	0.42	0.28	0.31	0.42	0.45	0.34	0.37	0.41	0.42	0.41	0.43	0.46	0.42	0.45	0.33

Computed correlation matrices are listed in Table 1
Estimated eigenvalues for each direction and the proportion of the total population variance explained by the PC's are listed in Table 2 below. The plots to the right visualise it graphically. They show that more than 50 percent of the total variance are explained by PC(1) for the north direction and a bit less in the other directions. More than 60 percent of the total variance are explained by the two first components for the north direction and a bit less for the other two directions. This indicates at least one or two dominating common effects or factors for all stations and directions.

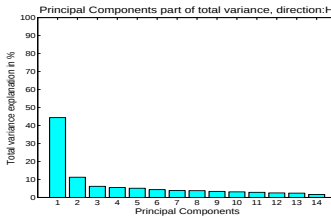
Direction: North



Direction: East



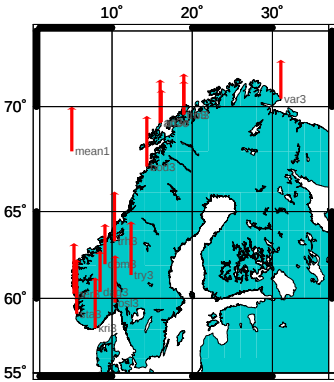
Direction: Height



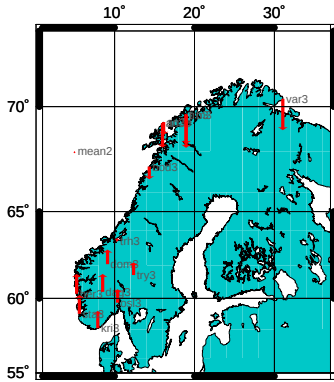
LOADINGS:

To show the contribution to the possible common effects from each station, the first three column vectors of the loadings, l_1 , l_2 and l_3 are computed. Their numerical values are listed in Table 2, and visualised as arrows of different lengths in the figures below, together with the mean value, or the average of the loadings, l_j -vector. The first factor loadings shows a common global variation in the whole country of Norway, almost uniform for all directions. The second loadings shows one or more regional effects which is different for the southern and northern part of the country for all directions. An axis from north-west to south-east north of Trondheim separates the regions. The third loadings may indicate a possible local effect or difference between stations in the mountain areas and stations close to sea level. This is particular visible in the height direction, but the few number of stations in the mountain area makes this determination uncertain.

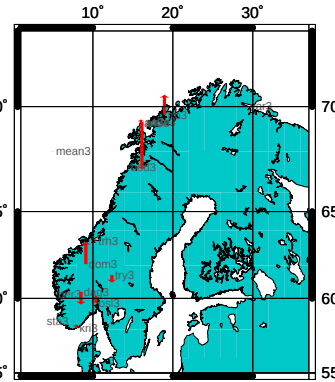
1. factor loadings (l_1)



2. factor loadings (l_2)



3. factor loadings (l_3)

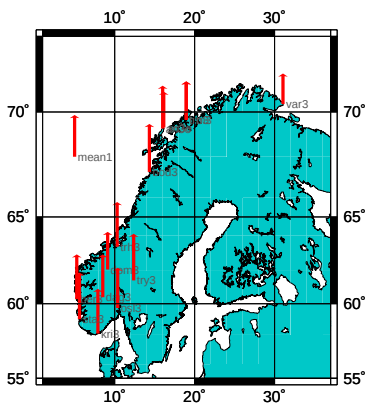


Direction: East

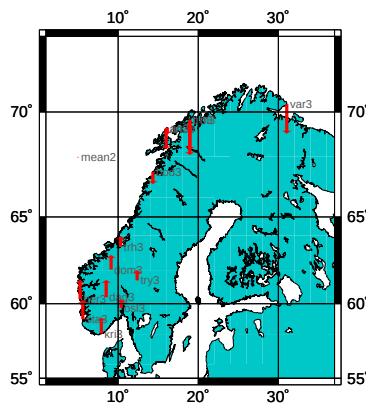
1. factor loadings (l_1)

2. factor loadings (l_2)

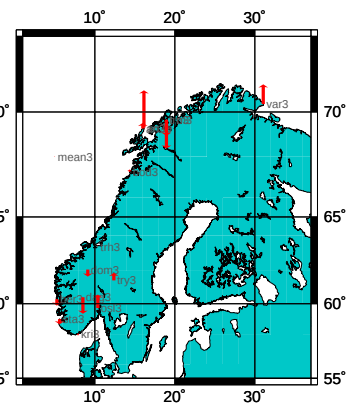
3. factor loadings (l_3)



1. factor loadings (I_1)



2. factor loadings (I_2)



3. factor loadings (I_3)

Table 2:

North direction			Columns of corresponding normalised eigenvectors														Factor loadings (Scaled normalised eigenvectors)			
Eigenvalues sorted by size, and their accumulated part of the total variance	value	acc.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	1. load.	2. load.	3. load.	
Lambda	1	7.044	0.593	0.194	-0.168	-0.084	0.158	-0.036	0.448	-0.045	-0.081	-0.036	-0.199	-0.044	-0.074	-0.082	-0.054	0.51404	-0.19972	-0.73718
Lambda	2	1.407	0.694	0.257	-0.317	0.067	0.250	-0.104	-0.461	-0.384	-0.181	-0.365	-0.449	-0.141	-0.166	0.054	-0.029	0.68341	-0.37583	0.06181
Lambda	3	0.840	0.664	0.296	0.259	0.018	-0.032	0.148	0.098	-0.040	-0.020	-0.482	0.216	-0.431	0.349	-0.473	0.014	0.78599	0.30745	0.01661
Lambda	4	0.694	0.713	0.299	-0.166	0.024	0.266	-0.130	0.012	-0.069	0.222	-0.151	0.231	0.744	0.268	-0.182	0.071	0.79297	-0.19647	0.02242
Lambda	5	0.633	0.758	0.241	0.235	-0.220	-0.066	-0.055	-0.273	0.056	0.276	0.058	-0.414	0.187	0.135	-0.092	0.069	0.04861	0.28925	-0.29158
Lambda	6	0.559	0.798	0.238	0.190	0.384	-0.052	0.634	-0.429	0.138	-0.242	-0.072	0.244	0.026	-0.041	0.063	-0.149	0.63226	0.22515	0.35172
Lambda	7	0.497	0.834	0.299	0.229	0.061	0.024	0.328	-0.043	0.059	-0.192	0.095	0.067	0.220	-0.652	-0.346	-0.313	0.79452	0.27119	0.05625
Lambda	8	0.461	0.867	0.277	0.177	0.119	0.356	0.403	-0.054	0.226	-0.223	0.329	-0.348	0.016	-0.463	-0.186	-0.080	0.73514	0.21050	0.09397
Lambda	9	0.392	0.895	0.319	0.244	-0.002	-0.036	0.110	0.054	-0.024	-0.114	-0.106	0.113	0.035	-0.241	0.362	0.771	0.04661	0.28925	-0.00162
Lambda	10	0.361	0.921	0.293	0.039	0.031	0.249	0.263	0.022	-0.192	0.457	0.582	0.058	-0.334	-0.093	-0.228	0.143	0.77679	0.04675	0.06176
Lambda	11	0.337	0.945	0.241	-0.384	0.000	-0.384	-0.021	-0.006	-0.332	-0.555	0.348	0.255	-0.011	0.189	-0.078	0.022	0.64015	-0.45522	0.00845
Lambda	12	0.309	0.967	0.199	-0.438	0.342	-0.230	0.413	0.458	-0.043	0.395	-0.076	-0.141	-0.067	-0.074	0.142	-0.064	0.52714	-0.51948	0.31388
Lambda	13	0.260	0.985	0.316	0.169	-0.129	-0.041	-0.066	-0.139	-0.101	0.161	-0.078	0.417	-0.098	0.024	0.063	-0.490	0.83768	0.20183	-0.31802
Lambda	14	0.205	1.000	0.231	-0.416	0.001	-0.033	-0.134	-0.283	0.780	-0.008	-0.036	0.163	-0.164	-0.073	-0.038	0.069	0.61263	-0.49316	0.00897
Sum Lambda		14.000																		

East direction			Columns of corresponding normalised eigenvectors														Factor loadings (Scaled normalised eigenvectors)			
Eigenvalues sorted by size, and their accumulated part of the total variance	value	acc.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	1. load.	2. load.	3. load.	
Lambda	1	5.987	0.428	0.238	-0.097	0.622	-0.079	-0.229	-0.584	0.067	0.091	0.168	-0.153	0.090	-0.018	0.154	0.234	0.56182	-0.11209	0.57966
Lambda	2	1.328	0.522	0.273	-0.274	0.099	0.004	0.093	-0.005	-0.254	-0.669	-0.361	0.029	-0.142	0.382	0.087	-0.124	0.60863	-0.31515	0.09246
Lambda	3	0.868	0.584	0.285	0.261	-0.107	0.096	0.097	-0.007	-0.229	-0.145	0.703	0.010	-0.223	0.036	0.341	-0.292	0.69771	0.30119	-0.09988
Lambda	4	0.784	0.640	0.305	-0.162	0.041	-0.010	0.060	-0.019	0.077	-0.156	0.034	0.751	0.076	-0.513	-0.092	0.049	0.74588	-0.18856	0.03796
Lambda	5	0.704	0.691	0.274	0.236	-0.280	-0.137	0.464	-0.189	0.035	-0.291	-0.158	-0.381	0.064	-0.308	0.060	0.462	0.67196	0.27237	-0.26109
Lambda	6	0.685	0.740	0.240	0.202	-0.114	-0.722	0.283	0.101	-0.207	-0.150	0.050	0.095	0.236	0.148	-0.289	0.196	0.58704	0.23274	-0.10602
Lambda	7	0.586	0.782	0.284	0.219	-0.030	0.218	0.014	-0.219	-0.138	0.444	-0.506	0.059	-0.444	-0.119	0.167	-0.243	0.69547	0.25220	-0.02793
Lambda	8	0.568	0.822	0.259	0.120	0.230	0.285	-0.406	0.575	-0.202	-0.026	-0.037	-0.286	0.087	-0.352	-0.004	0.100	0.63347	0.13793	0.21398
Lambda	9	0.547	0.861	0.308	0.261	-0.084	0.102	0.022	-0.156	-0.253	0.117	-0.067	0.040	0.718	0.209	-0.286	-0.274	0.73463	0.30125	-0.07785
Lambda	10	0.482	0.896	0.285	-0.146	0.026	-0.249	-0.131	0.188	0.750	-0.009	-0.100	-0.103	0.104	0.030	0.191	-0.302	0.60836	-0.10789	0.02425
Lambda	11	0.434	0.927	0.245	-0.478	-0.269	0.133	-0.121	-0.182	0.109	0.033	0.188	-0.343	-0.196	-0.175	-0.533	-0.234	0.59854	-0.55085	-0.25107
Lambda	12	0.413	0.956	0.243	-0.405	-0.407	0.059	-0.208	0.034	0.017	0.277	0.017	0.062	0.209	0.193	0.478	-0.317	0.59549	-0.46657	-0.45367
Lambda	13	0.357	0.982	0.297	0.135	0.127	0.387	0.241	0.157	0.314	0.083	0.104	0.152	-0.199	0.471	-0.395	0.389	0.72575	0.15586	0.11805
Lambda	14	0.259	1.000	0.197	-0.402	0.334	-0.268	0.575	0.349	-0.175	-0.008	-0.008	-0.123	0.061	0.014	0.074	-0.152	0.48266	-0.46368	0.31141
Sum Lambda		14.000																		

Height direction			Columns of corresponding normalised eigenvectors														Factor loadings (Scaled normalised eigenvectors)			
Eigenvalues sorted by size, and their accumulated part of the total variance	value	acc.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	1. load.	2. load.	3. load.	
Lambda	1	6.211	0.444	0.211	-0.232	-0.224	-0.762	0.105	0.336	0.270	-0.053	-0.124	0.197	-0.106	0.037	-0.068	-0.006	0.52660	-0.29131	-0.20811
Lambda	2	1.573	0.556	0.265	-0.277	-0.134	-0.023	0.351	-0.055	-0.266	-0.416	0.541	-0.395	-0.068	0.029	0.011	-0.101	0.66093	-0.34778	0.12470
Lambda	3	0.863	0.618	0.283	0.181	0.146	-0.184	-0.216	-0.101	-0.680	-0.033	-0.007	0.364	-0.139	-0.010	-0.391	-0.082	0.70629	0.22725	0.13577
Lambda	4	0.771	0.673	0.309	-0.201	-0.009	-0.162	0.018	-0.302	-0.117	0.176	-0.232	-0.174	0.684	-0.385	0.047	0.073	0.74886	-0.25185	-0.00866
Lambda	5	0.713	0.724	0.228	0.310	-0.486	0.204	-0.173	0.131	0.004	-0.484	-0.478	-0.237	-0.006	-0.014	-0.005	0.016	0.56929	-0.38928	-0.45112
Lambda	6	0.603	0.767	0.247	0.283	-0.413	0.049	-0.227	0.200	-0.028	0.515	0.410	-0.091	-0.072	-0.205	-0.141	-0.050	0.61524	-0.35471	-0.38349
Lambda	7	0.536	0.805	0.274	0.217	0.468	0.011	0.020	0.223	0.319	0.088	0.068	-0.399	0.051	0.030	-0.571	0.000	0.68294	0.27172	0.43499
Lambda	8	0.526	0.843	0.278	0.233	0.144	0.217	0.353	0.008	0.283	-0.284	0.127	0.529	0.033	-0.456	0.095	-0.036	0.69405	0.29226	0.13415
Lambda	9	0.468	0.876	0.274	0.243	0.466	-0.197	-0.108	0.150	-0.130	-0.087	-0.129	-0.198	-0.097	0.110	0.088	-0.023	0.68183	0.30505	0.43306
Lambda	10	0.432	0.907	0.290	0.029	-0.083	-0.037	0.150	0.030	0.202	0.259	-0.195	-0.134	-0.554	-0.021	0.003	-0.055	0.72321	0.03609	-0.07715
Lambda	11	0.396	0.935	0.268	-0.378	0.081	0.213	-0.451	-0.007	0.237	-0.053	0.028	0.108	0.060	0.119	0.007	-0.664	0.66700	-0.47448	0.07479
Lambda	12	0.348	0.960	0.277	-0.388	0.044	0.187	-0.393	-0.006	0.083	-0.097	0.109	0.125	-0.155	0.051	0.002	0.715	0.69071	-0.48704	0.04086
Lambda	13	0.332	0.984	0.301	0.244	-0.175	0.053	0.153	-0.160	0.100	0.109	0.148	0.229	0.304	0.724	0.053	0.110	0.75024	0.30542	-0.16272
Lambda	14	0.229	1.000	0.226	-0.324	0.018	0.380	0.441	-0.419	-0.258	0.325	-0.363	0.050	-0.113	0.097	0.006	-0.022	0.56216	-0.40675	0.01711
Sum Lambda		14.000																		

CONCLUSIONS:

The investigations of the CGPS-data have detected at least one or two common effects in the Norwegian SATREF-network. An attempt to interpret these effects is not carried out. A future modeling of these effects would probably decrease estimated variances for single-site processed CGPS-data with up to 50 %. A principal component analysis of CGPS-stations globally distributed all over the world would be of great interest.

REFERENCES:

- [Zumberge et. al., 1997]. J. F. Zumberge, M. B. Hefflin, D. C. Jefferson, and M. M. Watkins. Precise point positioning for the efficient and robust analysis of GPS data from large networks. *J. Geophys. Res.*, 102:5005–50017, 1997.
- [Haakonsen et. al., 2003]. Trond Arve Haakonsen, (Trond.Haakonsen@ntnu.no) Hossein Nahavandchi and Hans-Peter Plag. Determination of the Error-Spectrum of Continuous GPS Observations, presented in EGS-AGU-EUG Joint Assembly, Nice, 2003, see: http://www.geomatikk.ntnu.no/forskning/dring/poster_trond_Web.pdf

1) GPS Inferred Position SYstem - Orbit Analysis and Simulation II-software developed by NASA's Jet Propulsion Laboratory (JPL)